What you'll Learn About
- Definition of the derivative
- Notation

Definition of the derivative

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Use the substitution $h = x - a$ to create the definition of the derivative

A₁) Set-up a formula for the slope of $f(x) = x^2$ at $x = -1$

$$\lim_{x \to -1} \frac{(x)^2 - 1}{x + 1}$$

$$\lim_{h-1 \to -1} \frac{(h-1)^2 - 1}{h-1 + 1}$$

$a = -1$

A₂) Use the substitution $h = x - a$ to set-up the definition of the derivative

$h = x - (-1)$

$h = \Delta x$

$h = x + 1$

$(h - 1) = x$

$$\lim_{h \to 0} \frac{(h-1)^2 - 1}{h} = \frac{(h-1)(h-1) - 1}{h}$$

$$= \frac{h^2 - 2h + 1 - 1}{h}$$

$$= \frac{h^2 - 2h}{h} = \frac{h(h-2)}{h} = -2$$

B₁) Set-up a formula for the slope of $f(x) = \frac{1}{x-2}$ at $x = 4$

$$\lim_{x \to 4} \frac{\frac{1}{x-2} - \frac{1}{2}}{x - 4}$$

B₂) Use the substitution $h = x - a$ to set-up the definition of the derivative

$$\lim_{h+4 \to 4} \frac{\frac{1}{h+4-2} - \frac{1}{2}}{h+4-4} = \frac{\frac{1}{h+2} - \frac{1}{2}}{h}$$

$h = x - a$

$h = x - 4$

$\underset{+4 \quad +4}{}$

$h + 4 = x$

$$\lim_{h \to 0} \frac{\frac{1}{h+2} - \frac{1}{2}}{h}$$

Given a definition of the derivative(slope) find the function that you are taking the derivative of and the point you are finding the derivative(slope) at

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

A) $\displaystyle\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

$f(x) = \sqrt{x} \qquad x = 4$

→ Derivative of $\sqrt{x}$ at $x = 4$

B) $\displaystyle\lim_{x \to 2} \frac{\ln x - \ln 2}{x - 2}$

$f(x)$ ↓ (above $\ln x$)
↑ a (below $x \to 2$), ↑ a (below $x-2$)

$f(x) = \ln x \qquad x = 2$

0^x $\boxed{h = x - 2}$ ← $x = h + 2$

C) $\displaystyle\lim_{h \to 0} \frac{(2 + h)^3 - 8}{h}$

$f(x) = x^3 \qquad x = 2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

D) $\displaystyle\lim_{h \to 0} \frac{\frac{2}{3 + h} - \frac{2}{3}}{h}$

$f(x) = \frac{2}{x} \qquad \cancel{x = 3}$

$x = 3$

Another Definition: $\displaystyle\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$